Direct fragmentation of quarkonia including Fermi motion using light-cone wave function

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Abstract. We investigate the effect of Fermi motion on the direct fragmentation of the J/ψ and Υ states employing a light-cone wave function. Consistent with such a wave function we set up the kinematics of a heavy quark fragmenting into quarkonia such that the Fermi motion of the constituents splits into a longitudinal as well as a transverse direction and thus calculate the fragmentation functions for these states. In the framework of our investigation, we estimate that the fragmentation probabilities of J/ψ and Υ may increase at least up to 14 percent when including this degree of freedom.

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1 Introduction

Quarkonia production in high energy hadron collisions has been shown to be an important issue both in QCD and in collider physics. Hadronic production of J/ψ and Υ has received attention since the discovery of the charm and beauty quarks. The production of these states has been studied in interesting models [1]. It is also found that there is an order of magnitude discrepancy between cross section data and theoretical calculations in the case of J/ψ at the Tevatron energies. Then it became clear that the dominant mechanisms at high energy collisions are the color singlet [1,2] and the color octet [3] mechanisms and moreover that at the Tevatron energies the color singlet contribution is too small [4,5]. In the color singlet mechanism it is assumed that the $Q\bar{Q}$ pair is produced in a color singlet state at the production point whereas in the latter color octet case the $Q\bar{Q}$ pair is produced in the form of a color octet. Later on the emission of a soft gluon takes the pair into a color singlet state. To produce a singlet or octet $Q\bar{Q}$ there will of course be various channels out of which direct quark and gluon fragmentations are thought to be the dominant ones. In each case we need fragmentation functions to evaluate the probabilities and cross sections.

The very accurate and successful calculation of the fragmentation functions have recently been carried out in the limit of heavy quark effective theory. It has been proved that the strong interaction perturbation theory is suited to calculate the hard part of the heavy meson fragmentation in the above limit [6] and use a kind of wave function to introduce the soft smearing of the bound state.

These are then convoluted to give the fragmentation function. This convolution is possible only in a certain QCD scale. The Altarelli–Parisi evolution equation [7] takes this function into the required scale. In the case of fragmentation of heavy mesons, either a kind of delta function or the wave function at the origin of the bound state has been employed which produces a factor of meson decay constant, f_M, or the wave function at the origin $|R(0)|$, and lets the constituents move almost together in parallel ignoring their virtual motion. However the problem of the J/ψ cross section at Tevatron mentioned earlier and also its diffractive production [8] require a closer look at the process.

The common procedure for introducing the bound-state effects is to use the wave function at the center of momentum frame in a harmonic oscillator model. However the kinematics and parameterization in such a case are not very clear. On the other hand the procedure has successfully been studied in an infinite momentum frame. Brodsky, Huang and Lepage (BHL) [9] have proposed a boost free wave function for the heavy meson bound state which fits in the above calculational scheme. This is a kind of gaussian wave function with a parameter which adjusts its width; it is referred to as the confinement parameter. The mass of the constituents and their transverse momentum as well as the longitudinal energy momentum ratios appear in this function.

In this work we first describe the BHL wave function. Using it we calculate the fragmentation function for the J/ψ and Υ states in both cases i.e. when the Fermi motion is either ignored or not ignored. Change of the confinement parameter provides a nice comparison of these two

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functions. It is revealed that the inclusion of this degree of freedom may have a sizable effect on the fragmentation probabilities and hence on their cross section as measured in a particular machine aimed to produce such states.

2 Heavy meson wave function

The light-cone wave functions described in this section have satisfactorily been applied to light meson states such as π, ρ and K [10]. Motivated by these studies we will apply them to the case of J/ψ and Υ states.

The mesonic wave function can be obtained by solving the bound-state equation [11]

$$(M^2 - H_{\mathrm{LC}})|\mathcal{M}\rangle = 0, \qquad (1)$$

where H_{LC} is the hamiltonian in light-cone quantization and M is the mass of the meson $|\mathcal{M}\rangle$. In an attempt to solve this equation one expands $|\mathcal{M}\rangle$ in the complete set of Fock states,

$$|\mathcal{M}\rangle = \sum_{n,\lambda_i} \psi_n(x_i, \mathbf{q}_{\mathrm{T}_i}, \lambda_i)|n; x_i, \mathbf{q}_{\mathrm{T}_i}, \lambda_i\rangle. \qquad (2)$$

In the above relation $\psi_n(x_i, \mathbf{q}_{\mathrm{T}_i}, \lambda_i)$ $(i = 1, 2, ..., n)$ is the n-parton momentum-space amplitude defined on the free quark and gluon Fock basis in the light-cone formalism. The normalization condition is

$$\sum_{n,\lambda_i} \int [\mathrm{d}x][\mathrm{d}^2\mathbf{q}_{\mathrm{T}}]|\psi_n(x_i, \mathbf{q}_{\mathrm{T}_i}, \lambda_i)|^2 = 1, \qquad (3)$$

where

$$[\mathrm{d}x] \equiv \prod_{i=1}^{n} \mathrm{d}x_i \delta\left[1 - \sum_{i=1}^{n} x_i\right] \qquad (4)$$

and

$$[\mathrm{d}^2\mathbf{q}_{\mathrm{T}}] \equiv \prod_{i=1}^{n} \mathrm{d}^2\mathbf{q}_{\mathrm{T}_i} 16\pi^3 \delta^2\left[\sum_{i=1}^{n} \mathbf{q}_{\mathrm{T}_i}\right]. \qquad (5)$$

The sum is over all Fock states and helicities.

It is not easy to determine the form of the wave functions from the above expansion. The prescription due to BHL [9] provides a practical way by connecting the equal time wave function in the rest frame and the light-cone wave function, i.e. by equating the off-shell propagator $\epsilon = M^2 - \left(\sum_{i=1}^{n} q_i\right)^2$ in two frames:

$$\epsilon = \begin{cases} M^2 - \left[\sum_{i=1}^{n} p_i^0\right]^2 & [\mathrm{CM}], \\ M^2 - \sum_{i=1}^{n} \left[(\mathbf{q}_{\mathrm{T}_i}^2 + m_i^2)/x_i\right] & [\mathrm{LC}], \end{cases} \qquad (6)$$

where the constraints $\sum_{i=1}^{n} \mathbf{p}_i = 0$ and $\sum_{i=1}^{n} \mathbf{q}_{\mathrm{T}i} = 0$, $\sum_{i=1}^{n} x_i = 1$ apply for the CM and LC frames respectively. It is easily seen that for a two particle system with equal mass $(p_1^0 = p_2^0)$

$$\mathbf{p}^2 \longleftrightarrow \frac{\mathbf{q}_{\mathrm{T}}^2 + m^2}{4x(1-x)} - m^2, \qquad (7)$$

where p is the moment of the constituents in the center of momentum frame. Therefore the wave functions at two frames may be connected by

$$\psi_{\mathrm{CM}}(\mathbf{p}^2) \longleftrightarrow \psi_{\mathrm{LC}}\left[\frac{\mathbf{q}_{\mathrm{T}}^2 + m^2}{4x(1-x)} - m^2\right]. \qquad (8)$$

Now if we consider the wave function at the rest frame [12]:

$$\psi_{\mathrm{CM}}(\mathbf{p}^2) = A_M \exp\left[\frac{-p^2}{2\beta^2}\right], \qquad (9)$$

where β is the confinement parameter, and then using the connection (8), the IMF wave function in the case of equal mass constituents takes the following form:

$$\psi_M(x_i, \mathbf{q}_{\mathrm{T}}) = A_M \exp\left\{-\frac{1}{8\beta^2}\left[\frac{m_{\mathrm{T}}^2}{x_1} + \frac{m_{\mathrm{T}}^2}{x_2}\right]\right\}$$
$$= A_M \exp\left[\frac{m_{\mathrm{T}}^2}{8\beta^2 x_1(1-x_1)}\right]. \qquad (10)$$

Note that in this formalism $\mathbf{q}_{\mathrm{T}_1} = -\mathbf{q}_{\mathrm{T}_2}$; therefore $q_{\mathrm{T}_1}^2 = q_{\mathrm{T}_2}^2 = q_{\mathrm{T}}^2$. It is argued that this form holds in the case of heavy meson states with equal and unequal constituent quark and antiquark masses, i.e.

$$\psi_M(x_i, \mathbf{k}_{\mathrm{T}}) = A_M \exp\left\{-\frac{1}{8\beta^2}\left[\frac{m_{1\mathrm{T}}^2}{x_1} + \frac{m_{2\mathrm{T}}^2}{x_2}\right]\right\}, \qquad (11)$$

where $m_{1\mathrm{T}}^2 = m_1^2 + \mathbf{q}_{\mathrm{T}}^2$ and $m_{2\mathrm{T}}^2 = m_2^2 + \mathbf{q}_{\mathrm{T}}^2$. m_1 and m_2 are quark or antiquark masses chosen so that $m_1 \geq m_2$.

It is interesting to note that since the spin of the heavy quark decouples from the gluon field, the excited states have the same wave functions as the unexcited ones. The behavior of ψ_M^2 is shown in Fig. 1 for the J/ψ state with three different confinement parameters β which are employed to extract the effect of Fermi motion in this work. Naturally the shape of the wave function is very sensitive to the values of β. x_1 and x_2 both go from 0 to 1 with the constraint of $x_1 + x_2 = 1$. q_{T} should cover the necessary range to include the effective spreading of the wave function.

3 Kinematics

A typical lowest order Feynman diagram for fragmentation of a $c(b)$ quark into $J/\psi(\Upsilon)$ is shown in Fig. 2. To evaluate such a diagram consistent with the light-cone wave function for the bound state, we need to choose an infinite momentum frame in which the initial state heavy quark possesses transverse momentum of $\mathbf{p}'_{\mathrm{T}}$. This transverse momentum is to be carried by the final state $c(b)$ which produces a jet. The constituents of the meson after creation move along the z axis with a transverse momentum of $\mathbf{p}_{\mathrm{T}} = -\mathbf{k}_{\mathrm{T}}$ $(p_{\mathrm{T}} = k_{\mathrm{T}} = q_{\mathrm{T}})$. Therefore the particles' four momenta are taken as

$$p_\mu = [p_0, \ p_{\mathrm{L}}, \ \mathbf{q}_{\mathrm{T}}], \quad k_\mu = [k_0, \ k_{\mathrm{L}}, \ -\mathbf{q}_{\mathrm{T}}],$$
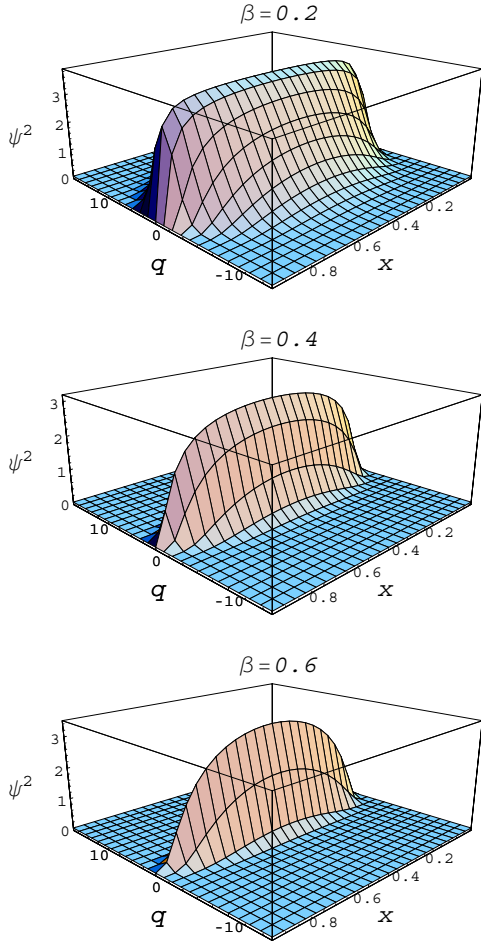
Fig. 1. The behavior of the BHL meson wave function (13) squared for the J/ψ state with different values of the confinement parameter. x is the longitudinal energy momentum ratio and q is the magnitude of the transverse momentum of the constituents

$$p'_\mu = [p'_0,\ p'_L,\ \mathbf{k}_T],\qquad k'_\mu = [k'_0,\ k'_L,\ \mathbf{k}_T]. \quad (12)$$

Note that all particles are moving in the forward direction and the q_T's are chosen in a manner consistent with the meson wave function. Moreover if we designate the meson four momentum by $\bar{p}$, then we may write $p_\mu = x_1\bar{p}_\mu$ and $k_\mu = x_2\bar{p}_\mu$ with the condition that $x_1 + x_2 = 1$. Therefore, we parameterize the energies of the particles by

$$p_0 = x_1 z p'_0,\ k_0 = x_2 z p'_0,\ k'_0 = (1-z)p'_0,\ p'_0 = p'_0, \ (13)$$

where we have used the definition of the fragmentation parameter $z = E_{\mathrm{hadron}}/E_{\mathrm{quark}}$ in an infinite momentum frame consistent with our kinematics.

4 The process of fragmentation

The very complex process of a heavy quark fragmenting into a heavy mesonic bound state is understood and becomes also calculable if we take into account the following.
(1) The hard part of the process of a heavy quark splitting into a heavy meson is calculable in the perturbative
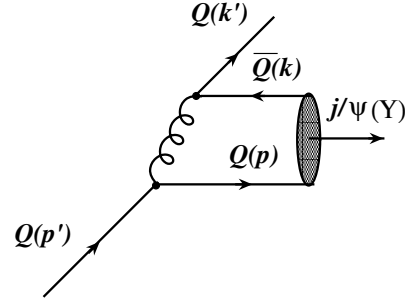

Fig. 2. The lowest order Feynman diagram for fragmentation of a heavy quark into a quarkonium state. The four momenta are labeled

QCD on the fragmentation scale [13]. The wave function of the bound state is convoluted to the hard scattering amplitude in an appropriate manner.
(2) To absorb the soft behavior of the bound state into the hard scattering amplitude a suitable wave function is employed (our choice (11)).

The hard scattering amplitude for the creation of the bound state may be put into the following form [14]:

$$T_H = \frac{4\pi\alpha_s m_1 m_2 M C_F}{2\sqrt{2p_0 p'_0 k_0 k'_0}}$$
$$\times \frac{\Gamma}{(k+k')^2(p_0+k_0+k'_0-p'_0)}. \quad (14)$$

Here $\alpha_s = g^2/4\pi$ is the strong interaction coupling constant, C_F is the color factor and Γ indicates that part of the amplitude which embeds spinors and gamma matrices. $(k+k')^2$ is due to the gluon propagator and $(p_0+k_0+k'_0-p'_0)$ is the energy denominator. In the case of Fig. 2, Γ has the following form:

$$\Gamma = \bar{u}(p)\gamma^\mu u(p')\bar{u}(k')\gamma_\mu v(k). \quad (15)$$

The initial spin average and final state spin sum of the squared Γ takes the following form:

$$\frac{1}{2}\sum_s \overline{\Gamma}\Gamma = \frac{1}{16m_1^2 m_2^2}\big[(p.k')(p'.k) + (p.k)(p'.k')$$
$$+ m_2^2(p.p') - m_1^2(k.k') - 2m_1^2 m_2^2\big]. \quad (16)$$

Regarding the kinematics introduced earlier, the dot products in the above read

$$2p.k' = \frac{1-z}{x_1 z}(m_1^2 + q_T^2) + \frac{x_1 z}{1-z}(m_2^2 + k_T^2)$$
$$- 2\mathbf{q}_T.\mathbf{k}_T, \quad (17)$$

$$2p'.k = \frac{1}{x_2 z}(m_2^2 + q_T^2) + \frac{x_2 z}{1}(m_1^2 + k_T^2) - 2\mathbf{q}_T.\mathbf{k}_T, \quad (18)$$

$$2p.k = \frac{x_1}{x_2}(m_2^2 + q_T^2) + \frac{x_2}{x_1}(m_1^2 + q_T^2) + 2q_T^2, \quad (19)$$

$$2p'.k' = \frac{1-z}{1}(m_1^2 + k_T^2) + \frac{1}{1-z}(m_2^2 + k_T^2) - 2k_T^2, \quad (20)$$

$$2k.k' = \frac{1-z}{x_2 z}(m_2^2 + q_T^2) + \frac{x_2 z}{1-z}(m_2^2 + k_T^2)$$
$$-2\boldsymbol{q}_T.\boldsymbol{k}_T, \tag{21}$$
$$2p'.p = \frac{1}{x_1 z}(m_1^2 + q_T^2) + \frac{x_1 z}{1}(m_1^2 + k_T^2) - 2\boldsymbol{q}_T.\boldsymbol{k}_T. \tag{22}$$

Here $\boldsymbol{q}_T.\boldsymbol{k}_T = q_T.k_T \cos\theta$ where θ is the angle between $\boldsymbol{q}_T$ and $\boldsymbol{k}_T$.

The fragmentation function of a heavy quark fragmentation into a heavy meson with wave function ψ_M is obtained from [14]:

$$D(z,\mu_0) \tag{23}$$
$$= \frac{1}{2}\sum_s \int d^3p\, d^3k\, d^3k'\, |T_H|^2 |\psi_M|^2 \delta^{(3)}(p + k + k' - p').$$

Here T_H is the hard scattering amplitude given by (14) and ψ_M is the meson wave function (11). Note that the average over the initial quark spin and a sum over the final state particles' spin are assumed. We substitute (14) into (23) to obtain

$$D(z,\mu_0) = \frac{(4\pi\alpha_s m_1 m_2 M C_F)^2}{8} \int \frac{d^3p\, d^3k\, d^3k'}{p_0 p_0' k_0 k_0'} \tag{24}$$
$$\times \frac{\frac{1}{2}\sum \overline{\Gamma}\Gamma |\psi_M|^2 \delta^{(3)}(p + k + k' - p')}{f(z)^2(p_0 + k_0 + k_0' - p_0')^2},$$

where $f(z) = (k + k')^2$. Now we perform the phase-space integrations. First of all note that

$$\int \frac{d^3p\, \delta^{(3)}(p + k + k' - p')}{p_0'(p_0 + k_0 + k_0' - p_0')^2} = \frac{p_0'}{g(z)^2}, \tag{25}$$

where $g(z) = (p + k + k')^2$. When the Fermi motion is ignored, $|\psi_M|^2$ is integrated with d^3k and gives

$$\int d^3k\, |\psi_M|^2 = \frac{z p_0'}{16\pi^3}, \tag{26}$$

where we have used the normalization condition (3) on the wave function. Also note that in this case we have

$$\int d^3k' = p_0' \int d^2k_T' = 2\pi p_0' \int k_T'\, dk_T'. \tag{27}$$

Instead of doing this integration, we simply replace k_T' by its average value. Thus we obtain

$$D(z,\mu_0,\beta=0) \tag{28}$$
$$= \frac{m_1(\alpha_s M C_F)^2 \langle k_T'^2 \rangle^{1/2}}{64 x_1 x_2} \frac{f'}{z(1-z)f(z)^2 g(z)^2},$$

where f' represents the square bracket in (16). When the Fermi motion is included, along the same line of calculation we obtain

$$D(z,\mu_0,\beta) \tag{29}$$
$$= \frac{m_1(\pi\alpha_s M C_F)^2 \langle k_T'^2 \rangle^{1/2}}{2} \int \frac{dq_T\, dx_2 q_T f' |\psi_M|^2}{x_1 x_2 z(1-z)f(z)^2 g(z)^2}.$$

Note that the above scheme of the phase-space integration allows one to compare (28) and (29) simply by varying the confinement parameter β. Further, note that we have distinguished the quark and antiquark masses such that (28) and (29) may be used in the case of other meson states such as D, B and B_c.

The explicit form of the fragmentation functions (28) and (29) in general are rather lengthy. However in the case of quarkonia they can be cast into rather brief forms. They appear in the appendix.

5 Results

We have shown that the wave function due to BHL may be employed to represent the soft behavior of the heavy meson bound state. In infinite momentum frame we have set up a kinematics which embed the Fermi motion of the constituents in both the longitudinal and transverse direction through this wave function whose width is controlled by the confinement parameter β. Comparison of (28) and (29) demonstrates the effect in an illustrative manner. This comparison is shown in Fig. 3 in the case of the J/ψ and Υ states. The two functions coincide at sufficiently small values of β. As we increase β, the peak due to
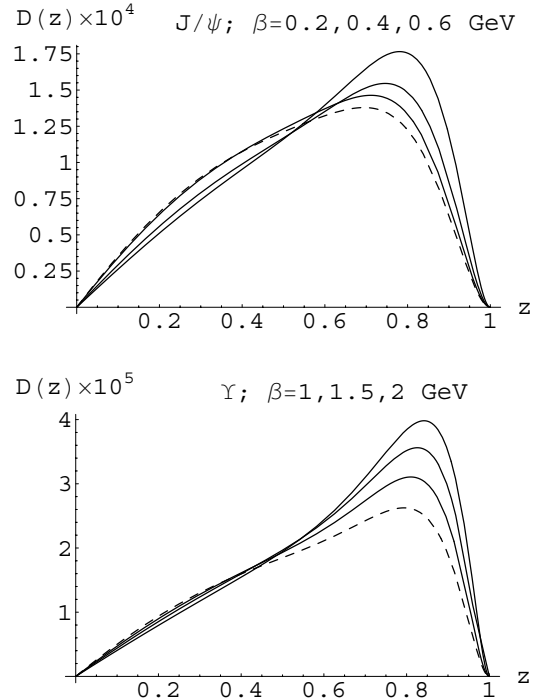
Fig. 3. The effect of the J/ψ and Υ wave functions on their fragmentations. While the dashed curve represent (28), the solid curves show the behavior of (29) with increasing values of β. The two curves coincide exactly for sufficiently small values of confinement parameter
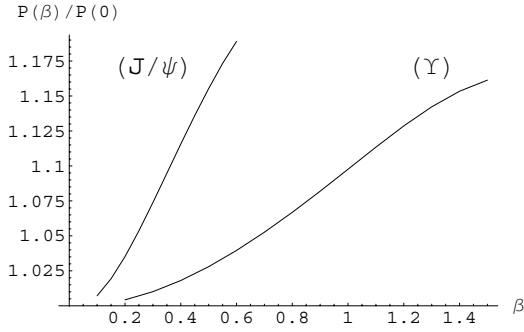
Fig. 4. Rate at which the fragmentation probabilities for J/ψ and Υ increase with increasing confinement parameter. Note the ranges of β selected for the two cases
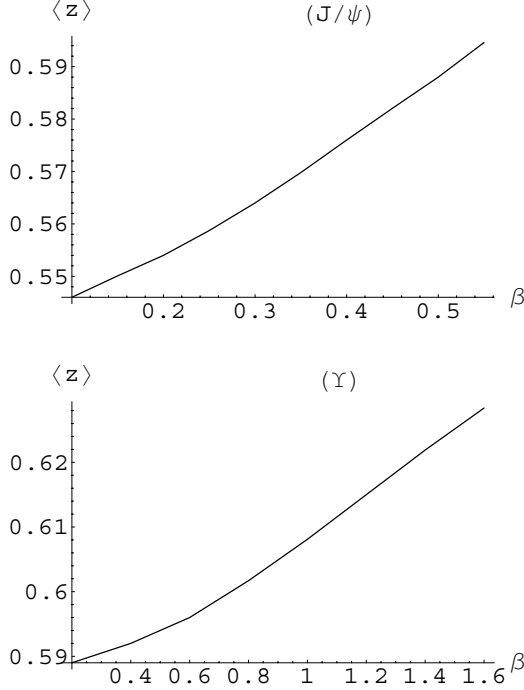
Fig. 5. Average fragmentation parameter $\langle z \rangle$ versus confinement parameter β for J/ψ and Υ

(28) rises and $\langle z \rangle$ moves to higher values. This behavior is consistent with the physics of the effect. In fact inclusion of Fermi motion in the fragmentation of a particular state means fragmentation of a state with higher energy and momentum which naturally ends up with higher values of both the fragmentation probability and average fragmentation parameter. Note the ranges of β selected in the two diagrams in this figure. The values for J/ψ are the same as those used in Fig. 1. To find the size of such changes, we employ the fragmentation probability defined by

$$P(\beta) = \int D(z, \mu_0, \beta) \mathrm{d}z \qquad (30)$$

and the average fragmentation parameter

$$\langle z \rangle(\beta) = \frac{\int z D(z, \mu_0, \beta) \mathrm{d}z}{\int D(z, \mu_0, \beta) \mathrm{d}z}. \qquad (31)$$

We plot $P(\beta)/P(0)$ versus β in Fig. 4. Such evaluations reveal that the Fermi motion of the constituents of J/ψ and Υ states improve the fragmentation probabilities of these states at least up to 14 percent. We have also plotted $\langle z \rangle(\beta)$ against β in Fig. 5.

This investigation shows that the Fermi motion within the heavy meson states such as J/ψ and Υ is indeed important and could not be ignored. This of course will be reflected on the cross section and event rate of these states at the colliders where they are expected to be produced.

Concerning the J/ψ production our results could be accounted for as a modification to the color singlet mechanism where the contribution due to various channels, specially the color octet mechanism, are relevant.

Appendix

The explicit form of (28) and (29) for quarkonia can be cast into the following form:

$$D(z, \mu_0, \beta = 0) = \frac{\alpha_s^2 C_F^2 \langle k_T'^2 \rangle^{1/2}}{16 m \mathcal{F}}$$
$$\times \left\{ z(1-z)^2 \left[\xi^2 z^4 + 2\xi z^2 (4 - 4z + 5z^2) \right. \right.$$
$$\left. \left. + (16 - 32z + 24z^2 - 8z^3 + 9z^4) \right] \right\},$$

and

$$D(z, \mu_0, \beta)$$
$$= \frac{\pi^2 \alpha_s^2 A_M^2 C_F^2 \langle k_T'^2 \rangle^{1/2}}{4m} \int \frac{\mathrm{d}q \mathrm{d}x q x^2 z (1-z)^2 |\psi_M|^2}{\mathcal{G}} \mathcal{H},$$

where $\mathcal{F}$, $\mathcal{G}$ and $\mathcal{H}$ read

$$\mathcal{F} = \left[\xi^2 z^4 - (z-2)^2 (3z-4) + \xi z^2 (8 - 7z + z^2) \right]^2,$$
$$\mathcal{G} = \left\{ \left[\eta(1-z)^2 + \xi x^2 z^2 + (1 - (1-x)z)^2 \right] \right.$$
$$\times \left[\eta(1-z) + \xi(1-x)xz^2 \right.$$
$$\left. \left. + (1 - (1 - x + x^2)z) \right] \right\}^2$$

and

$$\mathcal{H} = \left\{ 1 - 4(1-x)z + 2(4 - 10x + 7x^2)z^2 \right.$$
$$+ 4(-1 + 4x - 5x^2 + x^3)z^3$$
$$+ (1 - 4x + 8x^2 - 4x^3 + x^4)z^4$$
$$+ \eta \xi z^2 \left[1 - 2x + z^2 + x^2(2 - 2z + z^2) \right]$$
$$+ \left[\eta \left(2 - (4x - 6)z + (9 - 8x + 2x^2)z^2 \right. \right.$$
$$\left. - 2(2 - x + x^2)z^3 + (1 + x^2)z^4 \right)$$
$$+ \xi z^2 \left(1 + 2x^3(2 - 3z)z + z^2 + 2x^4 z^2 \right.$$

$$+ x(-2 + 2z - 4z^2) + x^2(2 - 8z + 9z^2)\Big)$$
$$+ \eta^2(1-z)^2 + \xi^2(1-x)^2 x^2 z^4 \Big] \Big\} \, .$$

Here $\xi = \langle k_{\mathrm{T}}'^2 \rangle / m^2$ and $\eta = q^2/m^2$. A_M is calculated using the normalization condition of the wave function, e.g. (3).

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